Updating and Forecasting with a Varying Parameter Recursive Model
UPDATING AND FORECASTING WITH A VARYING PARAMETER

RECURSIVE MODEL

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UPDATING AND FORECASTING WITH A VARYING PARAMETER RECURSIVE MODEL

J. Scott Shonkwiler

INTRODUCTION

Accurately forecasting prices of agricultural commodities during the present decade has been made difficult in light of severe shocks to the U.S. agricultural economy. The cattle sector has apparently undergone substantial disruption which was manifested, in part, by the sharp reduction of breeding cow inventories. In conjunction, quarterly average prices of Choice steers have displayed considerable variability during the 1970's. Specifically the run-up in Choice steer prices in the 1978-79 period has been unmatched by any other livestock price movements in recent history.

The present study develops a four equation recursive model capable of forecasting Choice steer prices two quarters ahead. The model admits a limited varying parameter structure in an effort to capture possible structural change. The varying parameter technique adopted permits reformulation of the model in terms of the Kalman filter time and measurement updating algorithms. Thus, updating the recursive model with recent data is handled systematically and forecast accuracy may be improved by weighting recent observations differently than the weighting that occurs

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when simply re-estimating an augmented observation set. The Kalman filter updating technique is developed for both the structural econometric model and its restricted reduced form. To assess the relative performance of this approach, a comparison of the forecasting accuracy of the two varying parameter models and their constant parameter counterpart is presented.

The following section outlines the varying parameter model, its implications and correspondence to a particular type of Kalman filter model and extends the varying parameter structure to it. Then, the subsequent sections present the specification and estimated parameters of the recursive model both under constant parameter and varying parameter regimes. Finally, the forecasting accuracy of the different models will be presented and discussed.

**The Varying Parameter Model**

The rationale for incorporating parameter variation stems from the lack of controlled effects and numerous unobservable forces inherent to modeling economic systems. Economists are typically constrained to analyzing secondary data with its attendant errors of reporting and collection with no assurance that the assumption of constant parameters holds unambiguously. The reasons for this uncertainty are twofold. First, the actual coefficients may be generated by an underlying non-stationary process. Or secondly, the true parameters may be stable within the appropriate or ideal model context but factors such as omitted variables, errors in variables, aggregation bias and improper functional form may preclude the formulation of the appropriate model. A varying parameter
specification may reduce the effects of these factors (Cooley and Prescott, 1973). Additionally, Cooley and Prescott (1976) have remarked that constant parameter formulations are inconsistent with theoretical specifications in the sense that the dynamics of economic behavior do not suggest constant-parameter behavioral equations.

The type of varying parameter structure adopted allows one or more coefficients to follow a first order Markov process. Specifically, the \( t \)-th observation for the model may be written

\[
y_t = x_t \beta_t + \epsilon_t \quad , \quad t = 1, 2, ..., T
\]  
(1)

\[
\beta_t = \beta_{t-1} + u_t
\]  
(2)

where \( x_t \) is a \( 1 \times k \) vector of observations on the independent variables, and \( \beta_t \) is a \( k \times 1 \) vector of coefficients at time \( t \). The stochastic assumptions for the model are

\[
E(\epsilon_t) = E(u_t) = E(\epsilon_s u_t) = 0
\]

(3a)

\[
E(\epsilon_s \epsilon_t) = \delta_{st} \sigma^2
\]

(3b)

\[
E(u_s u_t) = \delta_{st} Q
\]

(3c)

where \( Q \) is a \( k \times k \) covariance matrix assumed known and \( \delta_{st} \) denotes the Kronecker delta. The specification of \( Q \) is not as difficult as might seem. In the constant parameter case \( Q \) is identically equal to a null matrix. Otherwise, the variances and covariances may be specified in a manner similar to that used in mixed estimation (Cooper). That is, \( q_{ii} \) represents the variance of the varying parameter process of \( \beta_i \).
and $\beta_{it+2} \sqrt{\text{Var}}_{ii}$ would represent an approximate 95 percent confidence interval for the successive coefficient $\beta_{it+1}$.

Clearly estimation of the varying parameters must be referenced to some point during the sample period. Because interest is focused on forecasting, we desire the value of the parameters given the most recent observation. The values taken by the parameters given all observations through the most recent will be denoted $\beta_T$. To estimate $\beta_T$, Sant (1977) has formulated a generalized least squares model of the form

$$Y_T = X_T \beta_T + E_T - A_T U_T$$

$$\text{where}$$

$$A_T = \begin{bmatrix} x_1 & x_1 & \ldots & x_1 & x_1 \\ 0 & x_2 & \ldots & x_2 & x_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 & x_{T-1} \\ 0 & 0 & \ldots & 0 & 0 \end{bmatrix} \quad \text{and} \quad U_T = \begin{bmatrix} u_{2} \\ \vdots \\ u_{T-1} \\ 0 \end{bmatrix}$$

Alternatively (4) may be written compactly

$$Y_T = X_T \beta_T + V_T$$

Here $Y_T$ and $X_T$ represent the customary $T \times 1$ regressand vector and $T \times k$ design matrix. The complex structure incorporated into the disturbance vector $V_T$ results from the successive solution of (2) in terms of $\beta_T$. 
The matrix $A_T$ has dimension $T \times (T-1)k$ and $U_T$ is a $(T-1)k \times 1$ vector of parameter disturbances. The stochastic specification of $V_T$ is

$$\mathbb{E}(V_T) = 0$$

$$\mathbb{E}(V_T'V_T) = \sigma_e^2 I_T + A_T(I_{T-1} \Omega) A_T' = \Omega_T,$$  \hspace{1cm} (6a)  \hspace{1cm} (6b)

where $\sigma_e^2$ denotes the variance of the individual structural disturbances contained in $E_t$.

If $\Omega_T$ is known estimation of the parameter vector at time $T$ is given by

$$\hat{\beta}_T = (X_T' \Omega_T^{-1} X_T)^{-1} X_T' \Omega_T^{-1} Y_T.$$  \hspace{1cm} (7)

Of course, $\sigma_e^2$ is rarely known and cannot be estimated given $\hat{\beta}_T$ since $\Omega_T$ is conditioned by $\sigma_e^2$. Cooper has suggested an iterative procedure to calculate $\sigma_e^2$ which can be efficiently implemented. The method starts with an initial estimate of $\sigma_e^2$ and then derives $\hat{\Omega}_T$ and $\hat{\beta}_T$. Then a new estimate of $\sigma_e^2$ is given by

$$\hat{\sigma}_e^2 = (Y_T - X_T \hat{\beta}_T)' \hat{\Omega}_T^{-1} (Y_T - X_T \hat{\beta}_T) T^{-1}.$$  \hspace{1cm} (8)

and the procedure is repeated until convergence is achieved.

The forecasting and updating problem for the varying parameter model may be solved by using the appropriate Kalman filter recursions. The relationship between the Kalman filter model and generalized least squares models has been developed by Duncan and Horn and Sant (1977). The one step ahead prediction is given by the parameter time update

$$\hat{\beta}_{T+1/T} = \hat{\beta}_T.$$  \hspace{1cm} (9)
The evolution of the parameters may be referenced to previous parameter values and the new observations by the measurement update

$$\hat{\beta}_{T+1} = \hat{\beta}_T + K_{T+1}(y_{T+1} - x_{T+1}\hat{\beta}_T)$$

(10)

where $K$ represents the filter and is determined according to

$$K_{T+1} = \Sigma_{T+1}^{-1}x_{T+1}^\top \left[ x_{T+1}^\top \Sigma_{T+1}x_{T+1} + \frac{2}{c^\top} \right]^{-1}.$$ (11)

The matrix $\Sigma$ denotes the parameter covariance matrix. It is given sequentially by

$$\Sigma_{T+1|T} = \Sigma_T + Q$$

(12)

with measurement update

$$\Sigma_{T+1} = \Sigma_{T+1|T} - K_{T+1}x_{T+1}\Sigma_{T+1|T}.$$ (13)

For interpretation of the expressions (9)-(13) the interested reader is directed to Chow, Duncan and Horn, or Anderson and Moore. Implementation of the updating recursions in (9)-(13) is straightforward given the model developed in expressions (1)-(7). As stressed by Athans, the Kalman filter algorithm should not be confused with the underlying econometric model -- rather it should be viewed only as a means for refining a model.

The value of the Kalman filter model stems from the systematic manner in which new data may be incorporated. Expression (10) shows that as the forecast error increases, greater weight is given to the new observation in the determination of the updated parameters. However,
the filter adjusts this weighting according to equation (11) which contains an expression for the (inverse of the) forecast variance. That is, if the system indicates "large" forecast errors due to lack of resolution, then the forecast errors are weighted less because \( K \) is "smaller". However, if the forecast variance is (in some sense) small while the forecast error is large, then the updated parameter vector will depend much less on its previous value, i.e., the last observation will be weighted heavily.

The Varying Parameter Recursive Model

The extension of single-equation varying parameter techniques to simultaneous equation systems has been presented in several studies (Mariano and Schleicher, Narasimham et al., Mahajan and Mahajan), however, none of these studies showed the effect of a varying parameter structure on the evolution of the restricted reduced form. Further, it is not immediately clear whether the structure should be updated with predictions being formed from the restricted reduced form, or whether knowledge of the structure should be used to specify a restricted reduced form that can be updated directly. In view of the fact that combinations of restricted and unrestricted reduced forms (Maasoumi, Sant, 1978) may provide desirable properties, this latter approach will be developed as well.

The recursive simultaneous system represents a useful vehicle for development of the multiequation varying parameter model because the structural equations may be consistently estimated via immediate
application of ordinary least squares. Thus, questions associated with
the appropriate means to generate instrumental variables within a
varying parameter system are avoided. This section proceeds by first
briefly developing the general simultaneous equation model and the
recursive system in particular. This system is extended to admit varying
parameters and use of the updating algorithms.

Let the $T \times m$ matrix of $m$ jointly dependent variables and the $T \times 2$
matrix of predetermined variables be written

$$Y \Gamma + XB = \eta$$

where $\Gamma$ and $B$ are coefficient matrices of dimension $m \times m$ and $2 \times m$ re-
spectively and $\eta$ is a $T \times m$ matrix of structural disturbances. The
restricted reduced form of (14) is, of course.

$$Y = XP + \xi,$$

where

$$P = -B\Gamma^{-1} \text{ and } \xi = \eta\Gamma^{-1}.$$

The traditional stochastic assumptions concerning (14) and (15) include

$$E(\eta) = E(\xi) = 0$$

$$E(\eta') = \Phi$$

$$E(\xi'\xi) = \Gamma^{-1}\phi\Gamma^{-1}.$$

The matrix $\phi$ represents the structural covariances between the
disturbances of the $m$ equations in the system. In order to satisfy
the conditions of a truly recursive model, \( \Gamma \) is upper triangular and
\( \Phi \) is diagonal (Goldberger).

The recursive system may be conveniently written

\[
\text{Vec}(Y) = Z \Delta : \text{Vec}(\eta)
\]

(17)

where

\[
Z = \begin{bmatrix}
X_1 & 0 \\
\vdots & \ddots \\
Y_m & X_m
\end{bmatrix}
\quad \text{and} \quad
\Delta = \begin{bmatrix}
B_1 \\
B_2' \\
\vdots \\
B_m'
\end{bmatrix}
\]

The notation \( Y_i \) and \( X_i \) indicates the endogenous and predetermined
variables designated as regressors in the \( i^{th} \) equation. Estimation of
the structural parameters is given by

\[
\hat{\Delta} = (Z'Z)^{-1}Z'\text{Vec}(Y)
\]

(18)

with the variance of the estimated parameters in the \( i^{th} \) equation
determined as

\[
\text{Var}(\hat{\Delta}_i) = \Phi_i (Z'_iZ_i)^{-1} = \Psi_i
\]

(19)

If we let \( \Psi \) represent the block diagonal matrix whose \( i^{th} \) block is
\( \Psi_i \) then the variance of the restricted reduced form parameter may be
written as (Schmidt)

\[
\text{Var}(\hat{\eta}) = D\Psi W'D' = \Xi
\]

(20)
where

$$D = (\gamma^{-1})' \mathbf{W}_k$$

and \(\mathbf{W}\) represents the block diagonal matrix such that the \(i^{th}\) block of \(\mathbf{W}\) is

$$W_i = \text{plim} \ (X'X)^{-1}X'(Y_i';X_i).$$

The importance of obtaining the restricted reduced form parameters and their estimated variances will become clear when the varying parameter recursive model is set forth. The other measures developed are traditionally estimated and reported in most studies. The method adopted by Schmidt for obtaining \(\mathbf{Z}\) is particularly attractive due to its ease of implementation. This method, however, is based on the original Goldberger, et al. approach which is a Taylor series approximation based on large sample theory (Dhrymes).

To incorporate a first order Markov process as a parameter variation scheme, the recursive structural model may be written as

$$\text{Vec}(Y_T) = Z_T \Delta_T + \text{Vec}(n_T) - A_T^* \text{Vec}(U_T^*)$$

(21)

which introduces the structure of expression (4) to the system in (17). The matrix \(A_T^*\) is a block diagonal matrix whose \(i^{th}\) block would be \(A_{i1T}\) and would correspond to \(U_{i1T}\), the partition of \(U_T\) for the \(i^{th}\) equation. Again, the full model may be expressed

$$\text{Vec}(Y_T) = Z_T \Delta_T + V_T^*$$

(22)
with the stochastic specification that

\[ E(V_T^*) = 0 \quad (23a) \]

\[ E(V_T^* V_T^{*\prime}) = \Psi T + A_T^* (I_{T-1} \Psi Q^*) A_T^{*\prime} = \Omega_T^* \quad (23b) \]

In (23b) \( Q^* \) is a block diagonal matrix whose \( i^{th} \) block consists of the varying parameter covariance matrix of the \( i^{th} \) equation. Unless \( Q_i = 0 \), note that \( \Psi_{ii} \) in equation (23b) will differ from the same expression in (16b). Since \( \Omega_i^* \) is the sum of a diagonal and a block diagonal matrix, this result preserves the recursiveness of the system.

Estimation of the parameters at time \( T \) now follows from application of GLS to the system. Specifically

\[ \hat{\alpha}_T = (Z_T \Omega_T^{*^{-1}} Z_T^{*^{-1}})^{-1} Z_T \Omega_T^{*^{-1}} \text{Vec}(Y_T) \quad (24) \]

with the structural parameter covariance matrix for the \( i^{th} \) equation denoted by

\[ \text{Var}(\hat{\alpha}_{iT}) = (Z_{iT} \Omega_{iT}^{*} Z_{iT}^{*})^{-1} = \Psi_{iT}^* \quad (25) \]

Forecasts are generated from the reduced form implied by the varying structure. Thus, the reduced form parameters not only carry information concerning structural exclusion restrictions but also translate the varying parameter process from the structure to the reduced form. In general, the structural coefficients can be updated using the sequential algorithms presented in expression (9)-(13) and forecasts would be made using the updated restricted reduced form parameter matrix

\[ \hat{\Pi}_{T+1/T} = -\beta_{T+1/T} \hat{\alpha}_{T+1/T} \quad (26) \]
Alternatively the time varying structural model can be viewed as a convenient vehicle by which some appropriate structural specifications are employed to produce restrictions on both the parameter space and the parameter evolution process. The reduced form model implied by (22) is

$$Y_T = X_T \Pi_T + \xi_T^*$$

(27)
such that \(\hat{\Pi}_{t+1/T} = \hat{\Pi}_T\) and \(\xi_T^*\) represents the \(i^{th}\) column of \(\xi_T^*\) where

$$E(\xi_T^* \xi_T^*) = \iota^{-1} \phi_{T+1/T} \iota^{-1} + \Xi_T$$

(28)

The matrix \(\Xi_T\) represents a remainder term which is not required by the Kalman filter algorithm. Once (27) and (28) are given at time \(T\) additional information about the reduced form parameters is required in order to update expression (27) directly. Specifically the variance-covariance of the parameter \(\Pi_T\) and its evolutionary covariance must be determined. This first measure is derived by noting that

$$\text{Var}(\Pi_T) = \text{DW} \Psi_T \text{'DW}' = \Xi_T$$

(29)

Lastly the time update for the reduced form parameters' covariance follows directly as

$$\Xi_{T+1/T} = \Xi_T + \text{DWQ} \Psi_T \text{'DW}'$$

(30)

Expressions (27)-(30) permit updating the restricted reduced form directly. As additional sample observations are included it is expected that the restricted reduced form derived from the updated structure will diverge (26) from the directly updated reduced form. The rate of this divergence will be conditioned by numerous factors.
with a substantial influence possibly attributed to the compatibility of the restricted reduced form with the process generating the additional data. Remember that the structural parameters are not derived from minimizing the reduced form errors. The directly updated reduced form model, however, will be conditioned more by reduced form errors since the strength of the structural restrictions will deteriorate as additional measurement updates are made.

The Recursive Structural Model

Simultaneous equation models may offer advantages to least squares estimation of an unrestricted reduced form if the structural model provides useful restrictions that lead to more precise parameter estimates over the forecast interval. A problem typically hampering the use of many simultaneous equation models as forecasting tools is the requirement that many so-called predetermined variables must be forecast over the prediction period as well (Johnson). While methods are available for evaluating forecast variances for such models (Feldstein) they appear to weaken the structural approach particularly when the predetermined variables must be forecast for example using time series methods (Granger and Newbold).

In view of this, the recursive model specified uses predetermined variables which are either deterministic or lagged two or more quarters. The model relates quarterly Choice steer prices to current levels of fed and non-fed cattle slaughter and pork production. A trend variable is included to account for all other factors, particularly growth in consumer income given that fed beef is generally assumed to be a
superior good. Additional equations are required in order to predict levels of the livestock output variables two quarters ahead.

Non-fed slaughter is largely composed of cull cow slaughter and some grass-fed steers and heifers. Slaughter levels for this category typically increase when cow-calf operations are being reduced and when the price-cost outlook does not favor feeding out young animals. Thus, lagged prices for the fed product and feed costs (representing a major input) are expected to influence relative levels of non-fed slaughter. As fed steer prices fall and/or costs increase, non-fed cattle slaughter should increase.

Movements in the price and cost variables mentioned above are expected to have an opposite effect on fed cattle slaughter. Therefore, non-fed slaughter levels are hypothesized to be inversely related to slaughter of the fed category. In addition, variables which represent the number of feeder animals put on feed quarterly and seasonal patterns in fed cattle marketing are included. Although not all animals put on feed are automatically slaughtered within a fixed amount of time, lagged levels of this variable provide valuable information as to current fed slaughter levels.

For pork production, farrow to finish operations require about six months so lagged values of farrowings are good indicators of current production. Data are available on sow farrowings by quarter for the major pork producing states. Lagged prices provide a measure of the expected profitability foreseen by producers and, thus, should be positively correlated with current production.
Incorporation of these notions and an awareness of the seasonality inherent in the production side of the model led to the specification adopted. The four equation system is structurally recursive and the parameter matrix is triangular. The matrix of covariances between structural equations was assumed to be diagonal -- an assumption not particularly contradicted by the data given that the largest correlation between structural disturbance vectors was only .277 with 28 observations. Thus, the model is assumed to meet the theoretical conditions for a recursive system so that ordinary least squares may be applied to the estimation of the structure (Goldberger).

The model initially is estimated over the period 1971-I through 1977-IV. A longer period could have been chosen but it is likely that this would not benefit forecasting accuracy in view of the probable structural changes likely over even this short period. The structural specification and estimated constant parameters are presented in Table 1 and corresponding variable definitions may be found in the Appendix. Table 2 presents the derived reduced form and its estimated standard errors as well as similar estimates for the unrestricted reduced form.

The Varying Parameter Recursive Model and Forecast Performance

The parameter evolution structures adopted are presented in Table 1. The choice of coefficients was conditioned by the rationale that producer response to lagged prices may be subject to substantial variability over time. Thus, the non-fed cattle slaughter and pork production equations admit a plus or minus 8 unit and 1.4 unit change in
Table 1: Estimated structural parameters for constant and varying parameter specifications

<table>
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<th>Variable</th>
<th>Constant parameter structural models</th>
<th>Varying parameter models</th>
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<tr>
<td></td>
<td>NFCS</td>
<td>FC</td>
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<tr>
<td>NFCS</td>
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<td>FC</td>
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<tr>
<td>PORK</td>
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<td>SPR</td>
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<td>-1</td>
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<td>Q₂</td>
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</tr>
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<td>Q₃</td>
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<td>652</td>
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<tr>
<td>Q₂ Place₋₂</td>
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<tr>
<td>Q₃ Place₋₂</td>
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<td>Q₄ Place₋₂</td>
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<tr>
<td>s²</td>
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</tr>
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<td>a</td>
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Footnote: Figures in parentheses represent approximate t-values.
Table 2. — Derived and unrestricted reduced forms for choice steers prices

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<tr>
<th>Method</th>
<th>Restricted reduced form</th>
<th>Ordinary Least squares</th>
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<tbody>
<tr>
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<td>Constant Parameters</td>
<td>Varying Parameters</td>
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<tr>
<td>Variable</td>
<td>Parameter (Standard Error)</td>
<td>Parameter (Standard Error)</td>
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<tr>
<td>Intercept</td>
<td>98.4 (10.71)</td>
<td>88.07 (11.54)</td>
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<td>Q2</td>
<td>0.6367 (0.562)</td>
<td>0.2837 (0.319)</td>
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<tr>
<td>Q3</td>
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<tr>
<td>Q4</td>
<td>-2.147 (0.759)</td>
<td>-1.221 (0.707)</td>
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<tr>
<td>NFTS_{t-2}</td>
<td>-0.00109 (0.00037)</td>
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<tr>
<td>ZIPR_{t-1}</td>
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<td>0.1355 (0.0662)</td>
</tr>
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<td>-0.00268 (0.00111)</td>
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<td>Q3 * Place_{t-2}</td>
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<td>MAEG_{t-3}</td>
<td>-0.00266 (0.00117)</td>
<td>-0.00302 (0.00114)</td>
</tr>
<tr>
<td>FARR_{t-2}</td>
<td>-0.00466 (0.00114)</td>
<td>-0.00387 (0.00119)</td>
</tr>
<tr>
<td>Q2 * FARR_{t-2}</td>
<td>0.00103 (0.00040)</td>
<td>0.000849 (0.00034)</td>
</tr>
<tr>
<td>FARR_{t-3}</td>
<td>-0.00412 (0.00123)</td>
<td>-0.00338 (0.00121)</td>
</tr>
<tr>
<td>SNPR_{t-1}</td>
<td>-0.0385 (0.0245)</td>
<td>-0.0319 (0.0238)</td>
</tr>
<tr>
<td>Time</td>
<td>0.988 (0.0980)</td>
<td>0.463 (0.104)</td>
</tr>
</tbody>
</table>

\[ \chi^2 \] 12.83 7.43 16.47

DF 28 28 12
their respective price coefficients from period to period with 95 percent confidence in the limiting interval. These stochastic specifications are exhibited in the $\frac{\nu^2}{\mu}$ column of Table 1. Additionally, due to uncertainty on both the sign and the magnitude of the time coefficient in the Choice steer price equation it was specified to change by as much as plus or minus .2 with 95 percent confidence.

Interestingly, the structural results in Table 1 show little discrepancy between the constant parameter and varying parameter schemes. This may indicate that the variances attached to the varying parameters may be too small to effectively change the constant parameter values by very much. The restricted reduced forms for both models also reflect this similarity in coefficient magnitudes.

The contribution of the varying parameter specification adopted can be evaluated by the predictive performance of this method versus the constant parameter model. The predictive interval tests are presented in Table 3. It should be noted that the estimation of each model was based on data which occurred before the sharp run-up in Choice steer prices. Through the 1977-IV estimation period the highest price observed for was $48.64 during 1975-III. Table 3 separates the forecasts into two categories -- the one period ahead forecast based on parameter estimates made through the previous quarter, and the two period ahead forecast defined similarly. The "Naive" column corresponds to the structural model which is naively updated by adding additional unweighted observations successively. The "VARY-S" and "VARY RF" columns correspond to using the updating recursions for the structural model and the reduced form model, respectively. Of course,
Table 3.--Choice steer price predictive interval tests

<table>
<thead>
<tr>
<th>Forecast Period</th>
<th></th>
<th>Models estimated thru previous quarter</th>
<th></th>
<th>Models estimated two quarters previous</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual Value</td>
<td>Naïve</td>
<td>Vary-S</td>
<td>Vary-RF</td>
<td>Naïve</td>
</tr>
<tr>
<td>1978 I</td>
<td>45.77</td>
<td>39.72</td>
<td>42.10</td>
<td>42.10</td>
<td>41.13</td>
</tr>
<tr>
<td>1978 II</td>
<td>55.06</td>
<td>42.45</td>
<td>46.76</td>
<td>44.95</td>
<td>41.13</td>
</tr>
<tr>
<td>1978 III</td>
<td>53.75</td>
<td>46.18</td>
<td>53.76</td>
<td>52.24</td>
<td>43.29</td>
</tr>
<tr>
<td>1978 IV</td>
<td>54.76</td>
<td>46.20</td>
<td>51.60</td>
<td>52.85</td>
<td>44.93</td>
</tr>
<tr>
<td>1979 I</td>
<td>65.42</td>
<td>50.07</td>
<td>54.60</td>
<td>54.36</td>
<td>48.02</td>
</tr>
<tr>
<td>1979 II</td>
<td>72.51</td>
<td>57.75</td>
<td>63.56</td>
<td>64.32</td>
<td>54.21</td>
</tr>
<tr>
<td>1979 III</td>
<td>62.86</td>
<td>68.37</td>
<td>68.47</td>
<td>68.47</td>
<td>68.47</td>
</tr>
<tr>
<td>( \sigma_f )</td>
<td>11.40</td>
<td>6.95</td>
<td>7.20</td>
<td>14.41</td>
<td>11.14</td>
</tr>
<tr>
<td>MAE (^a)</td>
<td>10.82</td>
<td>5.82</td>
<td>6.08</td>
<td>13.98</td>
<td>9.85</td>
</tr>
</tbody>
</table>

\(^a\)MAE designates the mean absolute error of the forecasts.
the varying parameter structural model uses its restricted reduced form for prediction, but this reduced form is obtained by updating the structure -- not by updating the reduced form directly as in the second case.

The results in Table 3 indicate that the varying parameter approach achieves a respectable improvement in forecast accuracy over the constant parameter model. However, the substantial increase in the forecast errors from the one period ahead predictions to the two-period ahead case suggests that the variances on the varying parameters are not allowing sufficiently rapid adjustment of the parameters over the forecast interval. This is evident by considering the results in Table 3. Recall that for both forecast intervals the reduced form design matrix either consists of variables lagged two periods or deterministic components. Thus, the only difference between the one and two period ahead forecasts is the amount of information available with which to estimate the coefficients. Apparently the varying parameters are not adjusting rapidly enough to the new information conveyed by the measurement updates with the result that two periods ahead forecast accuracy suffers dramatically.

SUMMARY AND CONCLUSIONS

The varying parameter, generalized least squares, and Kalman filter models may all be related algebraically. By imposing a varying parameter structure on a behavioral relation, the corresponding filtering equation may be derived so as to permit efficient updating. A properly specified
varying parameter model should give increased forecast performance
(Athans). Anderson and Moore (p. 52) state that the traditional
constant parameter formulation with \( Q = 0 \) may not be wise since
"there is the possibility that the smallest of modeling errors can
lead, in time, to overwhelming errors" which were not predictable
from the estimated error covariance.

Within a multi-equation context the varying parameter structure
may be introduced. Given this framework updating may take place
with regard to the structural parameters or the (initially) re-
stricted reduced form parameters. The results for predicting quarterly
Choice steer prices indicate that both methods gave improved fore-
casting accuracy over the naively updated constant parameter model,
but with no clear advantage for either updating scheme evidenced.
APPENDIX
Appendix Table 1.--Variable definitions.

**Dependent variables**
- **FCS** - Fed cattle marketed, 23 states (1,000 head)
- **NFCS** - Non-fed cattle slaughter, equal to difference of total commercial cattle slaughter and FCS (1,000 head)
- **PORK** - Total commercial pork production (millions of pounds)
- **SPR** - Choice steer price, Omaha (dollars per cwt)

**Predetermined variables**
- Q1-Q4 - Quarterly dummy variables
- $ESPR_{t-i} = .2SPR_{t-2} + .5SPR_{t-3} + .3SPR_{t-4}$
- $ESFC_{t-i} = .2SFC_{t-2} + .5SFC_{t-3} + .3SFC_{t-4}$, where SFC is a steer feed cost index
- **Place** - Cattle placed on feed quarterly, 23 states (1,000 head)
- **Farr** - Sows farrowing, 14 states (1,000 head)
- $EHRP_{t-i} = .3HPR_{t-3} + .7HPR_{t-4}$, where HPR is the 7 market price of barrows and gilts (dollars per cwt)
- **Time** - Linear trend, 1971-I has value 1
FOOTNOTES

1. Numerous other varying, switching, or random parameter structures may be hypothesized, however, the present treatment is attractive because of its generality.

2. The stacking scheme presented simply permits the treatment of all equations jointly. Given a recursive system, single equation methods would yield identical results.

3. Alternatively, it could be assumed that the way in which expectations are formulated varies over time. Or, it could be argued that the model is mispecified in terms of incorporating expectations appropriately.
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